Chapter 37: Relativity

Group Members:

1. A spacecraft is flying at high-speed overhead as you stand on Earth. You see its searchlight blink on for a duration of $0.100s$. The captain of the spacecraft measures that the searchlight is on for a duration of $20.0ms$.

   a. Which of these two measured times is the proper time?

   The two on and off events for the searchlight occurs on the spacecraft so that the captain who is at rest with these two events will measure the proper time, $\Delta t_0 = 20.0ms$.

   b. What is the speed of the spacecraft relative to the Earth expressed as a fraction of the speed of light c (in v/c)?

   The Earth's observer measure the improper dilated time, $\Delta t = 0.100s$. Then, from the time dilation formula, we have,

$$\Delta t = \gamma \Delta t_0$$
$$\gamma = \frac{\Delta t}{\Delta t_0} = \frac{0.100s}{20.0 \times 10^{-3} s} = 5$$

   Then, from the definition of the gamma factor, we can solve for their relative speed,

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$
$$1-(v/c)^2 = (1/\gamma)^2$$
$$(v/c)^2 = 1-(1/\gamma)^2$$
$$\frac{v}{c} = \sqrt{1-(1/\gamma)^2} = \sqrt{1-(1/5)^2} = \sqrt{0.96} = 0.980$$

2. A spaceship at a space dock is measured to have a length of $100.0m$. It is then set onto a voyage toward a distance star. As it passes by the Jupiter station (assume that it has accelerated quickly and it has been traveling at a constant speed), an observer on Jupiter station measured its length to be $50.0m$. What is the speed of the spaceship? Express your answer as the ratio *v*/*c*.

The most important aspect for all these problems is to first identify which are the proper time and the proper length in the problem. For the proper length of a particular object, this special length interval must be measured by an observer stationary with the object.

In this case, the proper length of the spaceship is $100.0m$ measured at rest at the space dock. As the spaceship passing by Jupiter, the stationary observer on Jupiter measured a contracted length for the spaceship. To calculate the speed, we use the length contraction formula,

$$L = L_o / \gamma = L_o\sqrt{1 - v^2/c^2}$$

$$\left(\frac{L}{L_o}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v}{c} = \sqrt{1 - \left(\frac{L}{L_o}\right)^2} = \sqrt{1 - \left(\frac{50.0m}{100.0m}\right)^2} = \sqrt{0.750} = 0.866$$

$$v = 0.866c$$

3. A distance galaxy is about $1.00\times10^{5}\,light-years$ in diameter. The starship Enterprise enters the galaxy with a speed $v=0.990c$ as measured on Earth.

   a. How long does the Enterprise take to cross the galaxy from our viewpoint (stationary with respect to the galaxy)?

      From our viewpoint, the $1.00\times10^{5}\,light-years$ is the distance L which the Enterprise has to travel and its speed is 0.990c. Since the Earth is considered to be at rest with respect to the galaxy, L is the proper length L_o of the galaxy/ So, the time that it takes to cross the galaxy is,

      $$\Delta t=\frac{L_o}{v}=\frac{10^{5}\,light-years}{0.990c}=1.01\times10^{5}\,years$$

      Note that in using a physics equation such as above, all quantities must be measured by the same observer. In this case, all L_o, v, and t are measured with respect to you on Earth.

      Also, note that we have labeled the value $10^{5}\,light-years$ for the diameter of the galaxy as the proper length for the galaxy since we measure this length stationary with the galaxy.

   b. How long does the crew on the Enterprise think the journey takes?

      First we need to calculate the γ factor. With the relative speed between Earth and the Enterprise at $v=0.990c$,

      $$\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{1}{\sqrt{1-0.99^2}}=7.089$$

      The duration of the transit Δt as measured by the Earth's observer is NOT proper. Since the Enterprise crew are the ones who actually experienced the two events (entering and leaving the galaxy), the Enterprise crew measured time interval Δt_o is the proper time.

Then, from the time dilation formula, the Enterprise should
measure a short time interval as compared with the stationary observer.

$$\Delta t_o = \frac{\Delta t}{\gamma} = \frac{1.01\times 10^5\, years}{7.089} = 1.42\times 10^4\, years$$

c. How wide is the galaxy (along the direction of the motion) according to the crew of the Enterprise?

In the Enterprise's moving frame, the diameter of the galaxy along the direction of motion is Lorenz contracted, so using the Lorenz contraction formula, we have,

$$L = \frac{L_o}{\gamma} = \frac{10^5\, light-years}{7.089} = 1.41\times 10^4\, light-years$$

d. Now, using both the diameter as measured by the Enterprise (part c) and the standard distance/velocity = time formula to calculate the duration of passage as measured by the Enterprise. Check to see if your answer agrees with the one calculated in part b.

$L = 1.41\times 10^4\, light-years$ is a distance measured in the Enterprise frame.
Both the Earth (S frame) and the Enterprise (S' frame) measure the same relative speed except with opposite sign, so $v = 0.99c$ for the Enterprise crew also.

With respect to the Enterprise crew and using L and v measured by the Enterprise crew, the time that it takes to cross the contracted galaxy is,

$$\Delta t_o = \frac{L}{v} = \frac{1.41\times 10^4\, light-years}{0.990c} = 1.42\times 10^4\, years$$

This agrees with time dilation calculation in part b.